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Approximation hardness of deadline-TSP reoptimization[☆]

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ABSTRACT

Given an instance of an optimization problem together with an optimal solution, we consider the scenario in which this instance is modified locally. In graph problems, e. g., a singular edge might be removed or added, or an edge weight might be varied, etc. For a problem U and such a local modification operation, let LM-U (local-modification-U) denote the resulting problem. The question is whether it is possible to exploit the additional knowledge of an optimal solution to the original instance or not, i. e.,whether LM-U is computationally more tractable than U. While positive examples are known e.g. for metric TSP, we give some negative examples here: Metric TSP with deadlines (time windows), if a single deadline or the cost of a single edge is modified, exhibits the same lower bounds on the approximability in these local-modification versions as those currently known for the original problem.

1. Introduction

Traditionally, optimization theory has been concerned with the task of finding good feasible solutions to (practically relevant) input instances, little or nothing about which is known in advance. Many applications, however, demand good, sometimes optimal, solutions to a limited set of input instances which reflect a supposedly-constant environment (imagine, e. g., an existing railway system or communications network). When this environment does change, maybe only slightly and maybe only locally, do we have no choice but to recompute some good feasible solution, effectively forgetting about the old one?

Here, we will analyze *local* modifications only. In a graph problem, for example, the cost of a single edge might change, an edge might be removed or added, or some other local parameter might be adjusted. Results related to this work pertain to the question by how much a given instance of an optimization problem may be varied if it is desired that optimal solutions to the original instance retain their optimality [8,9,11–13]. In contrast with this so-called "postoptimality analysis," our approach here is to ask, if we cannot avoid losing the optimality of a given solution when an instance is varied arbitrarily, what can we do to *restore* the quality of a solution, maybe in an approximative sense?

Surely, for some problems, knowing an optimal solution to the original instance trivially makes their local-modification variants easy to approximate because the given optimal solution is itself a very good solution to the modified instance. For

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example, adding an edge in the instance of a coloring problem will increase the cost of an optimal solution by at most the amount of one — an excellent approximation, but certainly not the object of our interest.

This concept of local modification was investigated for metric TSP in [3], where modifications of edge costs were considered, and in [1,2], where the local modification consisted of vertex deletions and vertex insertions.

TSP with time windows is one of the fundamental problems in operations research [7]. In this generalization of TSP, certain vertices must be reached within a certain time window. Usually, only heuristic algorithms are used to attack it although the question how hard it is w. r. t. approximability has only recently been resolved in [5,4], where even an $\Omega(n)$ lower bound on the polynomial-time approximability of metric TSP (Δ-TSP for short) with time windows was shown, in contrast to the constant approximability of Δ-TSP. This lower bound already holds for the special case of this problem where all time windows are immediately open. This special case we will call metric TSP with deadlines, or Δ-DLTSP for short. Here, we consider local-modification versions of Δ-DLTSP. We show that, already if we only allow a single deadline to be changed, and only by an amount of one time unit, the resulting problem, LM(D)-Δ-DLTSP, has the same lower bound on the approximation ratio as Δ-DLTSP, namely that of asymptotically $n/2$. Let us underscore the importance of this negative result: Not only does TSP with deadlines remain an intractable problem in its local-modification version, but the extra knowledge of an optimal solution to a related instance does not even help a single bit. Likewise, we will establish a lower bound of $(2-\varepsilon)$, for any $\varepsilon > 0$, for LM(D)-Δ-DLTSP with a constant number of deadlines, the same bound as is known for Δ-DLTSP with a constant number of deadlines [5,4]. These results can also be obtained if we modify the cost of an edge rather than a deadline. Note that the original problem without reoptimization was proven to be approximable within a factor of 2.5 in [5,4].

So, while additional information about an optimal solution to a related input instance may be useful to some extent as shown e. g. in [3], the local-modification problem variant may remain exactly as hard as the original problem. Very recently, in [6] the reoptimization of Δ-DLTSP according to other local modifications such as addition and deletion of vertices was investigated.

The paper is subdivided into four main sections. In Section 2, we will formally define TSP with deadlines as well as its local-modification version. Section 3 is devoted to inapproximability results for the local-modification version of TSP with deadlines where there is a bounded number of deadline vertices, and Section 4 will deal with the general problem, where there may be an unbounded number of deadline vertices. In these two sections, we deal with the case where the local modification consists of changing a deadline. Finally, in Section 5, we analyze the case where the local modification consists of changing the costs of an edge.

2. Preliminaries

To begin with, let us define TSP with deadlines formally.

Definition 2.1. Let $G = (V, E)$ be a complete graph weighted by $c\colon E \to \mathbb{N}^+$. We call (s, D, d) a *deadline set* for G if $s \in V$, $D \subseteq V \setminus \{s\}$, and $d\colon D \to \mathbb{N}^+$. A vertex $v \in D$ is called *deadline vertex*. A path $(v_0, v_1, \ldots, v_n)$ *satisfies the deadlines* iff $s = v_0$ and, for all $v_i \in D$, we have $\sum_{j=1}^{i} c(\{v_{j-1}, v_j\}) \leq d(v_i)$.

A cycle $(v_0, v_1, \ldots, v_n, v_0)$ *satisfies the deadlines* iff it contains a path $(v_0, v_1, \ldots, v_n)$ satisfying the deadlines.

Definition 2.2. The problem Δ-DLTSP is defined as follows:
Input: A complete weighted graph $G = (V, E, c)$ with edge costs $c\colon E \to \mathbb{N}^+$ satisfying the Δ-inequality, a deadline set (s, D, d) for G, and a Hamiltonian cycle (of arbitrary cost) satisfying the deadlines.[1]
Problem: Find a minimum-cost Hamiltonian cycle satisfying all deadlines.

If $|D|$ is a constant k, the resulting subproblem is k-Δ-DLTSP.

For TSP with deadlines, at least two possible local modifications arise. Firstly, we can change the deadline of a deadline vertex, which leads to the following problem:

Definition 2.3. The optimization problem LM(D)-DLTSP is defined as follows:
Input: A complete weighted graph $G = (V, E, c)$, a deadline set $O = (s, D, d_O)$ for G with a minimal Hamiltonian cycle satisfying the deadlines O, a new deadline set $N = (s, D, d_N)$ such that d_O and d_N differ in exactly one vertex, and a Hamiltonian cycle (of arbitrary cost) satisfying N.
Problem: Find a minimum-cost Hamiltonian cycle satisfying N.

By LM(D)-k-DLTSP, LM(D)-Δ-DLTSP and LM(D)-k-Δ-DLTSP, we denote the canonical special cases of LM(D)-DLTSP.

The second local modification we consider here, is a change in the cost function for one edge.

[1] Requiring a feasible Hamiltonian cycle as part of the input ensures that the problem is in *NPO*. Otherwise, it would even be a hard problem to find a feasible solution. For details, see [5,4].

Definition 2.4. The optimization problem LM(E)-DLTSP is defined as follows:
Input: A complete weighted graph $G_O = (V, E, c_O)$, a deadline set (s, D, d) for G, with a minimum-cost Hamiltonian cycle satisfying the deadlines, and a complete weighted graph $G_N = (V, E, c_N)$ such that c_O and c_N differ in exactly one edge, and a Hamiltonian cycle (of arbitrary cost) in G_N satisfying the deadlines.
Problem: Find a minimum-cost Hamiltonian cycle in G_N satisfying the deadlines.

By LM(E)-k-DLTSP, LM(E)-Δ-DLTSP and LM(E)-k-Δ-DLTSP, we denote the canonical special cases of LM(E)-DLTSP.

For our proofs, we will need some reductions from the following two problems, which can easily be shown to be *NP*-hard analogously to the proof of the *NP*-hardness of the *restricted Hamiltonian cycle problem,* as presented, e.g., in [10].

Definition 2.5. The *restricted Hamiltonian path problem*, RHP for short, is the following decision problem:
Input: A graph $G = (V, E)$, two different vertices $s, t \in V$, and a given Hamiltonian path P from s to t.
Output: YES if G contains a Hamiltonian path starting in s, but ending in some vertex $v \neq t$, NO otherwise.

Definition 2.6. The *restricted Hamiltonian path problem with fixed endpoint*, FRHP for short, is the following decision problem:
Input: A graph $G = (V, E)$, three pairwise different vertices $s, t, x \in V$, and a given Hamiltonian path P from s to t.
Output: YES if G contains a Hamiltonian path starting in s and ending in x, NO otherwise.

3. Bounded number of deadline vertices

We start with the case where only few deadline vertices occur. Note that k-Δ-DLTSP can be approximated within a ratio of 2.5 [5,4]. Furthermore, a lower bound of $2 - \varepsilon$ on the approximability, for every small $\varepsilon > 0$, can be proved [5,4]. We will show that this lower bound also holds for LM(D)-k-Δ-DLTSP, even for $k = 4$.

For the proof we will distinguish two subproblems, namely increasing and decreasing the deadline of one deadline vertex. We will start with the increasing case.

Lemma 3.1. *Let $\varepsilon > 0$. There is no polynomial-time $(2 - \varepsilon)$-approximation algorithm for the subproblem of* LM(D)-2-Δ-DLTSP *where one deadline is increased by ξ time units, $1 \leq \xi < n$, unless $P = NP$.*

Proof. By means of a reduction, we will show that such an approximation algorithm could be used to solve FRHP. Let $\varepsilon > 0$.

Let (G', P) be an input instance for FRHP where $G' = (V', E')$, $|V'| = n + 1$, $s', t', x \in V'$, and P is a Hamiltonian path from s' to t'. Pick a $\gamma > \frac{5n+3}{2\varepsilon}$ (which implies $\frac{4\gamma+n+1}{2\gamma+3n+1} > 2 - \varepsilon$).

We construct a complete weighted graph $G = (V, E, c)$ as part of an input for LM(D)-2-Δ-DLTSP as shown in Fig. 1: We set $V := V' \dot{\cup} \{s, D_1, D_2\}$, and, for any edge e between two vertices $v_1, v_2 \in V'$, let $c(e) = 1$ if $e \in E'$ and $c(e) = 2$ otherwise. All edges depicted in Fig. 1 have the indicated costs while non-depicted edges obtain maximal possible costs that do not violate the triangle inequality. In particular, all vertices $v' \in G' \setminus \{s', t', x\}$ are connected to D_1, D_2, and s in the same way as the depicted vertex v. We set the deadlines $d(D_1) = \gamma + n$ and $d(D_2) = 2\gamma + n + 1$.

For these deadlines, one optimal solution $\overline{C}$ is the cycle $s, D_1, D_2, t', \ldots, s', s$, which uses the Hamiltonian path P backwards from t' to s' in G'. It costs exactly $\gamma + 1 + \gamma + \gamma + n + \gamma = 4\gamma + n + 1$.

Visiting D_1 and all vertices in G' before D_2 is impossible since it would violate the deadline at D_2 or D_1 in any case: On the one hand, any solution starting with s, D_1, v', for some $v' \in G'$, using some path in G' from v' to t' and then proceeding to D_2 can only satisfy the deadline at D_2 if it leaves out some vertex in G'. On the other hand, any solution starting with s, s' and some partial tour to x and then visiting D_1 before returning to G', also has to leave out some vertex from G' in order to obey the deadline at D_2. This is due to the fact that including D_1 in some tour through G' incurs an extra cost of at least 2. Visiting l still unvisited vertices in G' after D_2 incurs an additional cost of at least $2\gamma + l$; thus, a solution of this type cannot improve over $\overline{C}$.

Now, we increase $d(D_1)$ by ξ. If G' contains a Hamiltonian path P' from s' to x, a new optimal solution is s, P', D_1, D_2, s, and it costs $\gamma + n + 1 + \gamma + 2n = 2\gamma + 3n + 1$. If G' does not contain such a path, it is not possible to visit all vertices in G' before reaching D_1 and D_2. As $c(\{t', D_1\}) \geq 2$, we cannot follow the given Hamiltonian path P because this would violate the deadline $d(D_2)$. As in the old instance, including D_1 in some Hamiltonian path in G' would violate the deadline at D_2. Similar arguments hold for every other possibility. Hence, $\overline{C}$ remains an optimal solution in this case. Thus, we could use any approximation algorithm with an approximation guarantee better than

$$\frac{4\gamma + n + 1}{2\gamma + 3n + 1} > 2 - \varepsilon$$

to solve FRHP. This is why approximating this subproblem of LM(D)-2-Δ-DLTSP within $2 - \varepsilon$ is NP-hard. □

Lemma 3.2. *Let $\varepsilon > 0$. There is no polynomial-time $(2 - \varepsilon)$-approximation algorithm for the subproblem of* LM(D)-4-Δ-DLTSP *where one deadline is decreased by ξ time units, $1 \leq \xi < n$, unless $P = NP$.*

Proof. Let $\varepsilon > 0$. For the proof, we will use a reduction from RHP.

Let (G', P) be an input instance for RHP where $G' = (V', E')$, $|V'| = n + 1$, $s', t' \in V'$, and P is a Hamiltonian path from s' to t'. Pick some γ such that $\frac{4\gamma}{2\gamma+8n} > 2 - \varepsilon$.

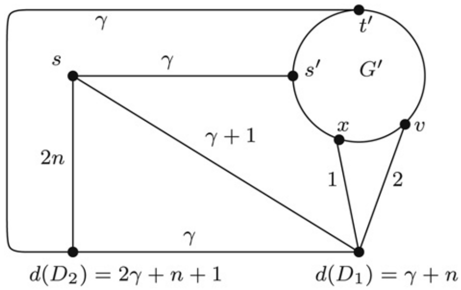

Fig. 1. Increasing a deadline. All vertices $v' \in V' \setminus \{s', t', x\}$ are connected like v.

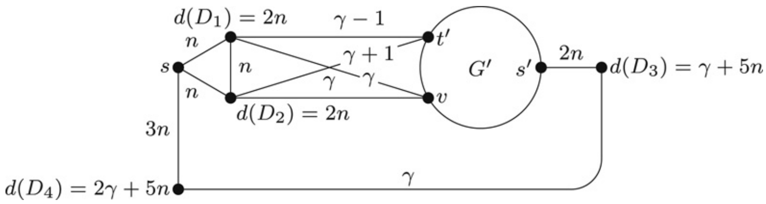

Fig. 2. Decreasing a deadline. All vertices $v' \in V' \setminus \{s', t'\}$ are connected like v.

We construct a complete weighted graph $G = (V, E, c)$ as part of an input for LM(D)-4-Δ-DLTSP as shown in Fig. 2: We set $V := V' \dot\cup \{s, D_1, D_2, D_3, D_4\}$, and, for any edge e between two vertices $v_1, v_2 \in V'$, let $c(e) = 1$ if $e \in E'$ and $c(e) = 2$ otherwise. All edges depicted in Fig. 2 have the indicated costs while non-depicted edges obtain maximal possible costs not violating the triangle inequality.

The initial deadlines are depicted in Fig. 2. In this setting, an optimal solution is the cycle $s, D_2, D_1, t', \ldots, s', D_3, D_4, s$, which contains the Hamiltonian path P from s' to t'. This path costs $2n + \gamma - 1$ on its way to G', spends n on the path from t' to s', and reaches s at time $2\gamma + 8n - 1$. Note that visiting D_1 before D_2 leads to an additional cost of at least 1 and is thus not optimal.

Now, we decrease the deadline $d(D_1)$ by ξ, whereby the old optimal solution becomes infeasible. Any new solution must visit D_1 before D_2. If we try to reuse the Hamiltonian path from t' to s', we have to spend $2n+\gamma+1$ on the way to t'. Therefore, we cannot reach D_3 if we follow the complete Hamiltonian path. Furthermore, we cannot visit any vertex $v \in V'$ between visiting D_3 and D_4 because D_3 is not reached before $4n + \gamma$, going back to V' would cost another $2n$, and the cheapest path from V' to D_4 costs more than γ. This is why any solution using a Hamiltonian path between s' and t' violates one of the deadlines $d(D_3)$, $d(D_4)$.

If G' contains a Hamiltonian path P from s' to some $v \neq t'$, the new optimal solution contains this path in reverse on its way to D_3. The path s, D_1, D_2, P, D_3, D_4 visits all vertices in V' between v and s' and reaches D_3 at time $\gamma + 5n$. Therefore, this new optimal solution costs $2\gamma + 8n$.

If G' does not contain such a Hamiltonian path, the optimal solution cannot visit all vertices in V' before reaching D_3 or even D_4, and consequently, it is more expensive than 4γ. Thus, we could use an approximation algorithm with an approximation guarantee better than

$$\frac{4\gamma}{2\gamma + 8n} > 2 - \varepsilon$$

to solve RHP. Hence, approximating this subproblem of LM(D)-4-Δ-DLTSP within $2 - \varepsilon$ is NP-hard. □

Lemmas 3.1 and 3.2 directly imply our main result of this section.

Theorem 3.3. *There is no polynomial-time* $(2 - \varepsilon)$*-approximation algorithm for* LM(D)-4-Δ-DLTSP, *unless* $P = NP$. □

4. Unbounded number of deadline vertices

When the number of deadline vertices is unbounded, we are able to show a linear lower bound on the approximability of LM(D)-Δ-DLTSP. Our reduction from RHP involves two steps. A first construction will guarantee that an optimal path becomes shorter by a constant factor if a Hamiltonian path exists in the RHP instance. A second construction inflates this advantage. Tours which start at time X, in contrast to those that start between times $X + g$ and $X + 2g$, may spend some extra time to visit a group of vertices which, unless visited early, will cause belated tours to run k times zigzag across a huge distance γ. Note that the construction will be such that tours starting after $X + 2g$ will violate some deadline in any case.

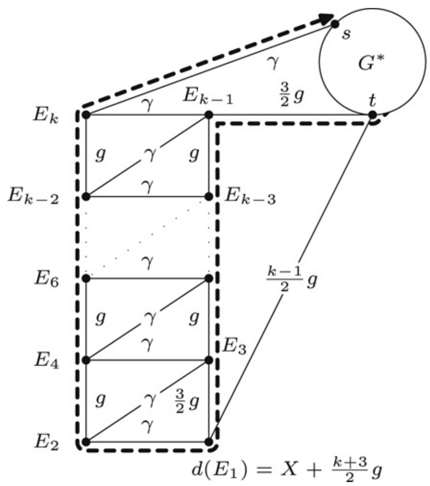

Fig. 3. The zigzag construction for the proof of Lemma 4.1. The left-hand side shows the optimal path if t is reached at time X. The right-hand side shows the optimal solution if t is reached after $X + g$. We set $d(E_{i+1}) := d(E_i) + \gamma$.

The following lemma describes the construction in detail. See Fig. 3 for an overview.

Lemma 4.1 (*Zigzag Lemma*). *Let $X, g, k, \gamma \in \mathbb{N}$ such that k is even and $\gamma \geq g$. Let $G^* = (V^*, E^*)$ be a complete graph with deadline set (s, D^*, d^*) such that any Hamiltonian path in G^* respecting the deadlines ends in the same vertex t. Then, we can construct a complete graph $G \supset G^*$ and deadlines (s, D, d) such that $D \supset D^*$, $d_{|D^*} = d^*$ and any path P in G^* that reaches t in time X can be extended to a Hamiltonian cycle $C = PP's$, where P' is a path visiting all vertices outside G^*, which costs at most*

$$X + kg + 2\gamma,$$

while any path that reaches t after $X + g$, but before $X + 2g$ can only be extended to a Hamiltonian cycle which costs at least

$$X + \frac{k+1}{2}g + k\gamma.$$

Proof. We construct $G = (V, E)$ with $V = V^* \cup \{E_1, \ldots E_k\}$ and edge costs as depicted in Fig. 3. To all other edges, we assign maximal possible costs that do not violate the triangle inequality. Note that the edge $\{t, E_1\}$ costs exactly the same as the path $E_{k-1}, E_{k-3}, \ldots, E_1$.

We set the deadlines

$$d(E_1) := X + \frac{k+3}{2}g \quad \text{and}$$
$$d(E_{i+1}) := d(E_i) + \gamma \quad \text{for all } i \in \{1, \ldots, k-1\}.$$

If a path reaches t strictly after $X + g$, it must proceed immediately to E_1. Note that it cannot use any other edge since it would have to use an edge of an additional cost of at least $\frac{3}{2}g$, then. Together with even the shortest path to E_1, this would violate this deadline. But then, it is forced to follow the sequence $E_2, E_3, \ldots, E_k$ to reach every deadline since even if it visited E_3 before E_2, it would incur an extra cost of $\frac{3}{2}g$, and this would violate the deadline of E_2. Analogously, proceeding from E_1, E_2 directly to E_4 would necessarily violate the deadline of E_3. Hence, the Hamiltonian cycle costs strictly more than $X + g + \frac{k-1}{2}g + k\gamma$.

A path that visits t before time X can visit $E_{k-1}, E_{k-3}, \ldots, E_3$ before E_1 because this path to E_1 costs at most

$$X + \frac{3}{2}g + \left(\frac{k}{2} - 2\right)g + \frac{3}{2}g = X + \frac{k+2}{2}g \leq d(E_1).$$

Closing the cycle to s, we obtain a cost of at most

$$X + \frac{k+2}{2}g + \left(\frac{k}{2} - 1\right)g + 2\gamma = X + kg + 2\gamma. \quad \square$$

We will now employ Lemma 4.1 to prove the desired lower bound. Again, we will consider the cases of increasing and decreasing a deadline separately.

Lemma 4.2. *Let $\varepsilon > 0$. There is no polynomial-time $\left(\left(\frac{1}{2} - \varepsilon\right) \cdot |V|\right)$-approximation algorithm for the subproblem of* LM(D)-Δ-DLTSP *where one deadline is increased by ξ for some $1 \leq \xi < n$, unless $P = NP$.*

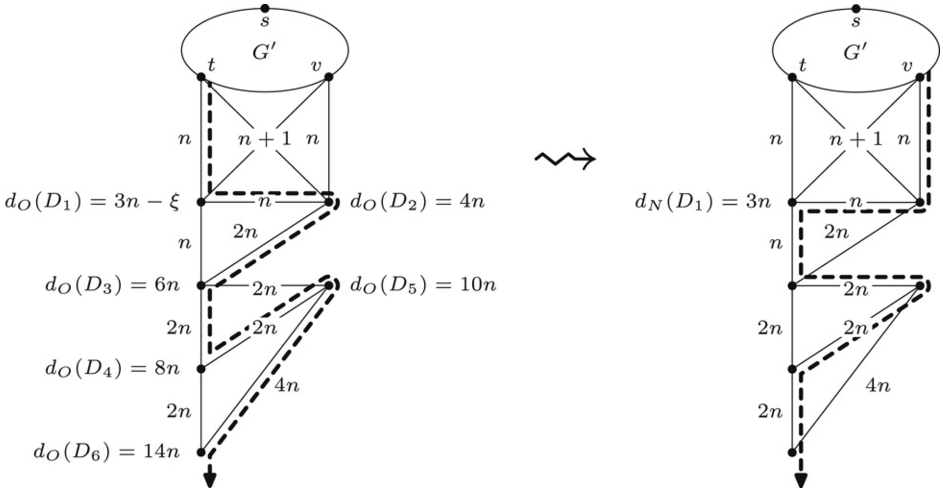

Fig. 4. Increasing a deadline: If the deadline for the vertex D_1 is increased, using a Hamiltonian path from s to v leads to a new optimal solution.

Proof. By means of a reduction, we will show that such an approximation algorithm could be used to solve RHP.

Let (G', P) be an input instance for RHP, where $G' = (V', E')$, $|V'| = n + 1$, $s, t \in V'$, and P is a Hamiltonian path from s to t.

We construct a complete weighted graph $G = (V, E, c)$ from G' as part of the input for the LM(D)-Δ-DLTSP instance as follows. The graph G contains a subgraph G^* as shown in Fig. 4 which is then complemented by the zigzag construction from Lemma 4.1.

We set $V^* = V' \cup \{D_1, \ldots, D_6\}$ and, for any edge e between two vertices $v_1, v_2 \in V'$, $c(e) = 1$, if $e \in E'$, and $c(e) = 2$ otherwise. To the other edges, we assign costs as depicted in Fig. 4, and maximal possible costs not violating the triangle inequality to the non-depicted edges; all edges from non-depicted vertices from G' to D_1 and D_2 get the same costs as the edges from v. We set the deadlines $d_O(D_i)$ according to Fig. 4. In particular, the vertex D_1 assumes a deadline of $3n - \xi$ for some $1 \le \xi < n$. Additionally, all vertices in G' assume a deadline of $2n$.

To construct the graph G_O of our desired LM(D)-Δ-DLTSP instance from G^*, we use the zigzag construction defined in Lemma 4.1 with parameters $X = 10n$, $g = 2n$, $k \ge (n+7)\frac{1-\varepsilon}{\varepsilon}$, and $\gamma \ge \frac{2kn+10n}{\varepsilon}$. This guarantees $2kn + 10n \le \varepsilon\gamma$ and $k \ge (1-\varepsilon)(k+n+7) = (1-\varepsilon) \cdot |V|$.

The given optimal Hamiltonian tour $\overline{C}$ in G starts in s, uses the given Hamiltonian path P in G' to t, and afterwards follows the sequence $D_1, D_2, D_3, D_4, D_5, D_6$ before entering the zigzag part of G. Hence, it reaches D_6 in time $13n$. It is easy to see that $\overline{C}$ is indeed optimal for the old deadline set: Any tour visiting D_2 before D_1 will violate the deadline at D_1. Following the zigzag construction, this leads to an overall cost of at least $10n + \frac{k+1}{2}g + k\gamma$.

Note that any path alternating between the zigzag part of G and G^* will necessarily violate the deadline of some yet-to-be-visited vertex E_i for some $i \in \{2, \ldots, k\}$.

In G_N, we change the deadline for D_1 from $3n - \xi$ to $d_N(D_1) = 3n$. The tour $\overline{C}$ remains a feasible solution. If G' contains a Hamiltonian path from s to some vertex $v \neq t$, an optimal solution uses this path and follows the sequence $D_2, D_1, D_3, D_5, D_4, D_6$. This solution reaches D_6 in time $10n$. By Lemma 4.1, this cycle has overall costs $10n + kg + 2\gamma$.

If G' does not contain any Hamiltonian path to such a vertex v, then $\overline{C}$ remains the optimal solution. By Lemma 4.1, we again obtain a cost of $10n + \frac{k+1}{2}g + k\gamma$. Together with our choices of the parameters, this leads to a ratio of at least

$$\frac{10n + \frac{k+1}{2} \cdot 2n + k\gamma}{10n + k \cdot 2n + 2\gamma} > \frac{k\gamma}{2kn + 10n + 2\gamma}$$
$$> \frac{k\gamma}{(2+\varepsilon)\gamma} = \frac{k}{2+\varepsilon} \ge \frac{1-\varepsilon}{2+\varepsilon}(k+n+7) \ge \left(\frac{1}{2} - \varepsilon\right)|V|.$$

Hence, a polynomial-time $(\frac{1}{2} - \varepsilon)|V|$-approximation algorithm could be used to solve RHP. □

Lemma 4.3. *Let $\varepsilon > 0$. There is no polynomial-time $\left(\left(\frac{1}{2} - \varepsilon\right)|V|\right)$-approximation algorithm for the subproblem of* LM(D)-Δ-DLTSP *where one deadline is decreased by $\xi \ge 1$ unless $P = NP$.*

Proof. By means of a reduction, we will show that such an approximation algorithm could be used to solve the RHP. Let (G', P) be an input instance for RHP, where $G' = (V', E')$, $|V'| = n + 1$, $s, t \in V'$, and P is a Hamiltonian path from s to t. We construct a complete weighted graph $G = (V, E, c)$ as part of an input for the LM(D)-Δ-DLTSP as as follows. The graph G contains a subgraph G^* as shown in Fig. 5 which is then complemented by the zigzag construction from Lemma 4.1.

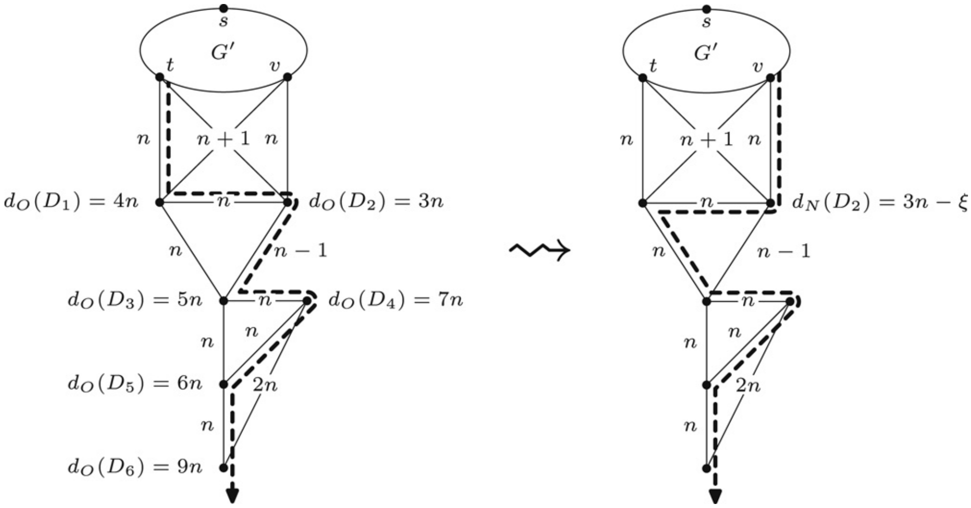

Fig. 5. Decreasing a deadline: If the deadline for the vertex D_2 is decreased, the old optimal solution (depicted on the left side) is not feasible anymore. If G' contains a Hamiltonian path from s to v, we obtain the depicted new optimal solution. If no such Hamiltonian path exists, the new optimal solution must follow $D_2, D_1, D_3, D_5, D_4, D_6$.

We set $V^* = V' \cup \{D_1, \dots, D_6\}$ and, for any edge e between two vertices $v_1, v_2 \in V'$, $c(e) = 1$, if $e \in E'$, and $c(e) = 2$ otherwise. For the other edges, we assign the costs as depicted in Fig. 5, and maximal possible costs not violating the triangle inequality to the non-depicted edges, and we set the deadlines $d_O(D_i)$ according to Fig. 5.

Again, we use Lemma 4.1 to extend the graph G^*, using the parameters $X = 7n - 1$, $g = n$, $k \geq (n+7)\frac{1-\varepsilon}{\varepsilon}$, and $\gamma \geq \frac{(k+7)n}{\varepsilon}$. The given optimal solution $\overline{C}$ uses the Hamiltonian path from s to t and visits $D_1, D_2, D_3, D_4, D_5, D_6$ afterwards. D_6 is reached at time $7n - 1$, therefore the optimal Hamiltonian cycle $\overline{C}$ costs at most $7n - 1 + kn + 2\gamma$.

Note that any path alternating between the zigzag part of G and G^* will necessarily violate the deadline of some yet-to-be-visited vertex E_i for some $i \in \{2, \dots, k\}$.

In G_N, we change the deadline for D_2 as follows: $d_N(D_2) = 3n - \xi$ for some $1 \leq \xi < n$. The cycle $\overline{C}$ is no feasible solution anymore since it visits D_2 at time $3n$. If G' contains a Hamiltonian path from s to some vertex $v \neq t$, an optimal solution for G_N uses this path and follows the sequence $D_2, D_1, D_3, D_4, D_5, D_6$. It reaches D_7 in time $7n$. Hence, by Lemma 4.1, extending it to a Hamiltonian cycle costs at most

$$7n + kn + 2\gamma.$$

If G' does not contain a Hamiltonian path to any vertex $v \neq t$, an optimal solution will visit all vertices in V' and then D_2. This vertex is not reached before $2n + 1$. Thus, following the cheap path D_2, D_1, D_3, D_4, D_5, the deadline of vertex D_5 is violated. Hence the solution must visit the remaining vertices in the order given by D_1, D_3, D_5, D_4, D_6. It does not reach D_6 before $8n + 1$, hence by Lemma 4.1 the corresponding Hamiltonian cycle costs at least

$$7n + \frac{k+1}{2}n + k\gamma.$$

We obtain the ratio

$$\begin{aligned}\frac{7n + \frac{k+1}{2}n + k\gamma}{7n + kn + 2\gamma} &> \frac{k\gamma}{(7+k)n + 2\gamma} \geq \frac{k\gamma}{(2+\varepsilon)\gamma} = \frac{k}{2+\varepsilon} \\ &\geq \frac{1-\varepsilon}{2+\varepsilon}(k+n+7) = \frac{1-\varepsilon}{2+\varepsilon}|V| \geq \left(\frac{1}{2} - \varepsilon\right)|V|.\end{aligned}$$

Hence, a polynomial-time $(\frac{1}{2} - \varepsilon)|V|$-approximation algorithm could be used to solve RHP. □

From Lemmas 4.2 and 4.3 we can directly conclude the following:

Theorem 4.4. *Let $\varepsilon > 0$. There is no polynomial-time $\left((\frac{1}{2} - \varepsilon)|V|\right)$-approximation algorithm for* LM(D)-Δ-DLTSP, *unless* $P = NP$. □

5. Modifying edge costs

In the previous sections, we have analyzed deadline modifications. In what follows, we will show similar results for local modifications of edge costs. We start with the case of TSP with only a bounded number of deadlines.

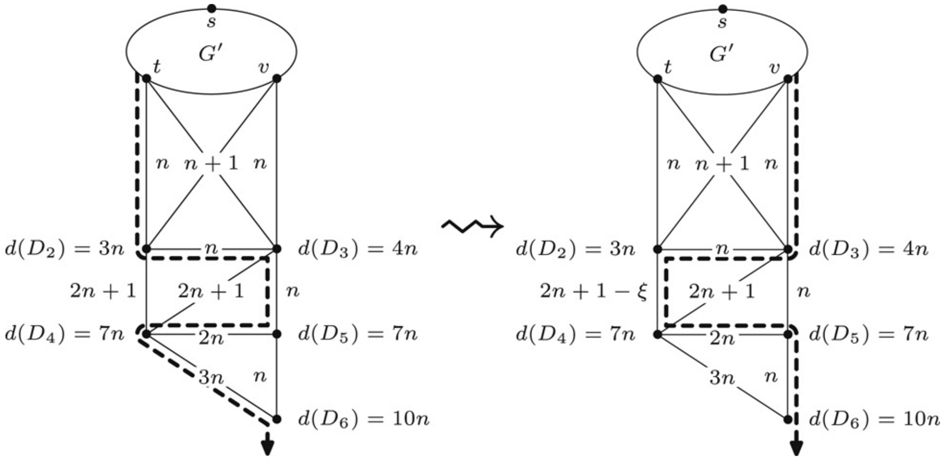

Fig. 6. Decreasing edge costs: If the cost of the edge $\{D_2, D_4\}$ is decreased, using a Hamiltonian path from s to v leads to a new optimal solution.

Theorem 5.1. *Let $\varepsilon > 0$. There is no polynomial-time $(2-\varepsilon)$-approximation algorithm for* LM(E)-2-Δ-DLTSP *even if only the cost of one edge is changed, unless P = NP.*

Proof. Let $1 \le \alpha < n$. In the case of increasing edge costs, we use the same construction as in Fig. 2. Decreasing the deadline $d(D_1)$ by α can be simulated by increasing the edge cost of $\{s, D_2\}$ by α. Both approaches guarantee that the previous optimal solution is not feasible anymore. The remainder of the proof is then identical to the proof of Lemma 3.2.

In the case of decreasing edge costs, we reuse the construction from Fig. 1. Here, the deadline $d(D_1)$ is set to $\gamma + n + 1$, the edge $e = \{D_1, D_2\}$ now costs $\gamma + 1$. In this setting, an optimal solution is still $s, D_1, D_2, t', \ldots, s', s$. Decreasing the costs of e by α admits the new optimal solution $s, s', \ldots, x, D_1, D_2, s$ iff G' contains a Hamiltonian path from s to x. The remainder of the proof is identical to the proof of Lemma 3.1. □

Also, for an unbounded number of deadlines, we are able to establish the same lower bounds as for deadline modifications.

Theorem 5.2. *Let $\varepsilon > 0$. There is no polynomial-time $\left((\frac{1}{2} - \varepsilon)|V|\right)$-approximation algorithm for* LM(E)-Δ-DLTSP *even if only the cost of one edge is changed by some ξ where $1 \le \xi < n$, unless P = NP.*

Proof. The case of increasing edge costs again is similar to decreasing a deadline. We use the left graph from Fig. 5. If the costs of the edge $\{t, D_1\}$ increase by some $\xi \ge 1$, the old optimal solution is not feasible anymore. Hence, the same argument as in Lemma 4.3 can be applied.

In the case of decreasing edge costs, we will show, by means of a reduction, that such an approximation algorithm could be used to solve the RHP. We use a similar construction as in the proofs before, see Fig. 6 for an overview. Let (G', P) be an input instance for RHP, where $G' = (V', E')$, $|V'| = n + 1$, $s, t \in V'$, and P is a Hamiltonian path from s to t.

Any solution in the unmodified graph has to visit the vertices in V', D_2 and D_3 before any other deadline. Note that the path $s, \ldots, t, D_2, D_3, D_4, D_5$ violates the deadline of D_5. Therefore, an optimal solution starts with $s, \ldots, t, D_2, D_3, D_5, D_4, D_6$. It reaches D_6 in time $9n$. Now, we decrease the costs of $\{D_2, D_4\}$ by $1 \le \xi < n$. Iff G' contains a Hamiltonian path from s to some other vertex v, a new optimal solution is $s, \ldots, v, D_3, D_2, D_4, D_5, D_6$ and costs $8n + 1 - \xi$. Otherwise, a new optimal solution has to visit D_2 before D_3, leading to the old path $s, \ldots, t, D_2, D_3, D_5, D_4, D_6$ with costs $9n$. Applying the construction from Lemma 4.1 with parameters $X = 8n$ and $g = n$ and choosing k and γ analogously to the previous proofs, we obtain the desired gap between these two cases. □

6. Conclusion

In this work, we have shown that the concept of reusing optimal solutions when input instances are locally modified cannot be used to reduce the complexity of TSP with deadlines. This problem is remarkably hard [5,4], and we have been able to reestablish almost all known lower bounds on the approximability of its variants in the setting of local modifications.

As an open problem, we state the question how hard it is to approximate locally modified TSP with deadlines in the near-metric case, where only the relaxed triangle inequality $c(u, v) \le \beta(c(u, z) + c(z, v))$ holds for any triple u, v, z of vertices and for some $\beta > 1$.

References

[1] C. Archetti, L. Bertazzi, M.G. Speranza, Reoptimizing the traveling salesman problem, Networks 42 (3) (2003) 154–159.

[2] G. Ausiello, B. Escoffier, J. Monnot, V.Th. Paschos, Reoptimization of minimum and maximum traveling salesman's tours, in: Proc. of the 10th Scandinavian Workshop on Algorithm Theory, SWAT '06, in: Lecture Notes in Computer Science, vol. 4059, Springer-Verlag, 2006, pp. 196–207.

[3] H.-J. Böckenhauer, L. Forlizzi, J. Hromkovič, J. Kneis, J. Kupke, G. Proietti, P. Widmayer, Reusing optimal TSP solutions for locally modified input instances (extended abstract), in: Proc. of the 4th IFIP International Conference on Theoretical Computer Science, Springer-Verlag, 2006, pp. 251–270.
[4] H.-J. Böckenhauer, J. Hromkovič, J. Kneis, J. Kupke, On the approximation hardness of some generalizations of TSP, in: Proc. of the 10th Scandinavian Workshop on Algorithm Theory, SWAT '06, in: Lecture Notes in Computer Science, vol. 4059, Springer-Verlag, 2006, pp. 184–195.
[5] H.-J. Böckenhauer, J. Hromkovič, J. Kneis, J. Kupke, On the parameterized approximability of TSP with deadlines, Theory of Computing Systems 41 (3) (2007) 431–444.
[6] H.-J. Böckenhauer, D. Komm, Reoptimization of the metric deadline TSP, in: Proc. of the 33rd International Symposium on Mathematical Foundations of Computer Science, MFCS 2008, in: Lecture Notes in Computer Science, vol. 5162, Springer-Verlag, 2008, pp. 156–167.
[7] J.-F. Cordeau, G. Desaulniers, J. Desrosiers, M.M. Solomon, F. Soumis, VRP with time windows, in: P. Toth, D. Vigo (Eds.), The Vehicle Routing Problem, 2001, pp. 157–193.
[8] H. Greenberg, An annotated bibliography for post-solution analysis in mixed integer and combinatorial optimization, in: Advances in Computational and Stochastic Optimization, Logic Programming, and Heuristic Search, ISBN: 0-7923-8078-9, 1998, pp. 97–147.
[9] S. van Hoesel, A. Wagelmans, On the complexity of postoptimality analysis of 0/1 programs, Discrete Applied Mathematics 91 (1999) 251–263.
[10] J. Hromkovič, Algorithmics for Hard Problems. Introduction to Combinatorial Optimization, Randomization, Approximation, and Heuristics, Springer-Verlag, 2003.
[11] M. Libura, Sensitivity analysis for minimum Hamiltonian path and traveling salesman problems, Discrete Applied Mathematics 30 (1991) 197–211.
[12] M. Libura, E.S. van der Poort, G. Sierksma, J.A.A. van der Veen, Stability aspects of the traveling salesman problem based on k-best solutions, Discrete Applied Mathematics 87 (1998) 159–185.
[13] Y.N. Sotskov, V.K. Leontev, E.N. Gordeev, Some concepts of stability analysis in combinatorial optimization, Discrete Applied Mathematics 58 (1995) 169–190.